REDUCTION OF GAS BREAKDOWN VOLTAGE WITH PULSED IONIZING RADIATION

A. A. Dantser and V. A. Feoktistov

UDC 537,56

An algorithm is described for computer calculation of the dynamic breakdown voltage of a gas gap affected by a spatially uniform pulse of ionizing radiation. The algorithm is based on numerical integration of a system of nonlinear equations with integral boundary conditions. The program is used to calculate the breakdown voltage of an air gap affected by a bell-shaped ionizing pulse. It is shown that the relative reduction in breakdown voltage can amount to tens of percent for a radiation exposure dose rate $P_0 \sim 10^8$ R/sec.

1. Charged particles are produced in a gas as the result of the action of external radiation. If the amplitude of the pulse is sufficiently large, the resultant space charge distorts the initial uniform electric field (in the case of a plane-parallel electrode system) so that the development of electron avalanches takes place in a nonuniform field. If the original parameters E_0 , p, and d (E_0 is the initial electric field, p the pressure, and d the distance between electrodes) are such that deviation of the field from uniformity as a result of the effect of space charge leads to an increase in the multiplication factor μ , the electrical stability of a gas ionized by an external source will decrease.

Estimates of the effect of an external steady-state ionization of a gas on the breakdown voltage have been made [1-3] by applying perturbation theory to the time-independent system of equations including the Poisson equation. These results are restricted because of the limitation of a small relative change in the breakdown voltage.

A study of the transient discharge current when space charge is neglected has been made by various methods [4, 5]. Inclusion of the Poisson equation in a time-dependent system of equations makes analytic solution difficult. An approximate method of solution was developed [6-8] which made it possible to study the asymptotic variation of the current, including the effects of space charge, for $t \sim T_+ \gg T_-$ (where T_+ and T_- are the flight times across the interelectrode gap for ions and electrons respectively). The method assumes that the space charge appears during a time $\sim T_+$ as the result of a buildup of positive ions during the transit of a large number of electron avalanches. As was demonstrated [6, p. 142], the transition to a self-sustaining discharge can occur at an initial value of the multiplication factor μ_0 less than but sufficiently close to one, ensuring the transit of a large number of avalanches before the space charge mechanism begins to take effect and the factor μ becomes greater than one.

Here we consider the transient discharge current in a gap affected by a powerful ionizing pulse of the form $Q(t) = Q_0 f(t)$ having a duration $T \sim T_- (Q_0)$ is the peak value of the number of charges produced by the external source per unit volume and per unit time; the value of Q_0 in CGS units for air at normal pressure agrees numerically with the exposure dose rate P_0 expressed in roentgens per second). The amplitude Q_0 is such that space charge formed during the pulse distorts the applied field, i.e.,

$$dTQ_0 \gg U / 4\pi d \tag{1.1}$$

where U is the potential difference.

This leads to a need to consider the effect of space charge for times t ~ T_. Since the effect of the external radiation in this case leads to strong distortion of the field and a change in the factor μ , the tran-

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 52-60, November-December, 1973. Original article submitted December 25, 1972.

© 1975 Plenum Publishing Corporation, 227 West 17th Street, New York, N.Y. 10011. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, microfilming, recording or otherwise, without written permission of the publisher. A copy of this article is available from the publisher for \$15.00.

sition to a self-sustaining discharge may occur for values of the factor μ_0 markedly less than one.

An algorithm and program for the calculation of the breakdown voltage of a gas located in a field of pulsed ionizing radiation are developed here from an analysis of [9, 10], which are devoted to the application of numerical methods to the solution of certain problems in gas-discharge physics, and a quantitative evaluation of the effect is made for specific cases.

2. We consider a plane-parallel electrode system with the x axis directed from cathode to anode. An ionization pulse of the form Q(t) is incident on the gas gap. The initial equation system and boundary conditions have the form

$$\partial q_{j} \partial t = \alpha (E) j_{j} - \partial j_{j} \partial x + Q (t)$$
(2.1)

$$\partial q_{+} / \partial t = \alpha (E) j_{-} + \partial j_{+} / \partial x + Q (t)$$

$$\partial E / \partial x = 4\pi (g - g_{-})$$
(2.2)

$$\partial E/\partial x = 4\pi \left(q_{-} - q_{+}\right) \tag{2.3}$$

$$j_{-}(0, t) = \gamma_{i} j_{+}(0, t) + \gamma_{*} \int_{0}^{1} \alpha(E) j_{-}(x, t) dx$$
(2.4)

$$j_{+}(d, t) = 0$$

Here, q_{-} and q_{+} are the charge densities for electrons and positive ions, j_{-} and j_{+} are the current densities for electrons and positive ions, $\alpha(E)$ is the impact ionization coefficient, E is the electric field, and γ_i and γ_* are the coefficients of secondary ionization at the cathode for ion impact and photoeffect.

The Poisson equation (2.3) must be solved under the condition

$$\int_{0}^{d} E(x, t) dx = U(t)$$

One can obtain a differential equation for the potential U(t) containing parameters of the external circuit and of the discharge gap by applying Kirchhoff's laws to the electrical circuit. We assume the potential across the discharge gap is kept constant:

$$\int_{0}^{a} E(x, t) \, dx = U = \text{const}$$

Considering Eq. (2.3), the boundary condition for the field takes the form

$$E(0, t) = \frac{U}{d} - \frac{4\pi}{d} \int_{0}^{d} dx \int_{0}^{x} [q_{-}(x', t) - q_{-}(x', t)] dx'$$
(2.5)

Problems in the numerical calculation are the determination of the transient current J(t) in the external circuit

$$J(t) = \frac{1}{d} \int_{0}^{d} [j_{-}(x', t) + j_{+}(x', t)] dx'$$
(2.6)

and the determination of the differential field distribution E(x, t) at different times. If the potential across the discharge gap is not kept constant, the current in the external circuit is given by

$$J(t) = \frac{1}{d} \left\{ \frac{1}{4\pi} \frac{dU}{dt} + \int_{0}^{a} [j_{-}(x, t) + j_{+}(x, t)] dx \right\}$$
(2.7)

In the calculations, the following empirical expressions were used for the quantities v_, v_+, and α : 1) for air,

$$a(E) = \begin{cases} C_1 p \exp[-D_1 E/p], & E/p < W \\ C_2 p \exp[-D_2 p/E], & E/p > W \end{cases}$$

$$v_- = \mu_- E/p, \quad v_+ = \mu_+ E/p \qquad (2.8)$$

where $\mu_{-} = 4 \cdot 10^5 \text{ cm}^2 \cdot \text{mm Hg/V} \cdot \text{sec}$, $\mu_{+} = 2 \cdot 10^3 \text{ cm}^2 \cdot \text{mm Hg/V} \cdot \text{sec}$, $C_1 = 2.67 \cdot 10^{-8} (\text{cm} \cdot \text{mm Hg})^{-1}$, $C_2 = 8 (\text{cm} \cdot \text{mm Hg})^{-1}$, $D_1 = -0.35 \text{ mm Hg} \cdot \text{cm/V}$, $D_2 = 247 \text{ V/cm} \cdot \text{mm Hg}$, and $W = 35 \text{ V/cm} \cdot \text{mm Hg}$. Equations (2.8) give a satisfactory approximation to tabulated data [11, p. 75] over the range 20 < E/p < 200;

2)) for argon

$$v_{+} = \begin{cases} (\mu_{+}E/p) (1 - B_{1}E/p), & E/p < W' \\ (k_{+}\sqrt{E}/\sqrt{p}) (1 - B_{2}(p/E)^{\gamma_{2}}), & E/p > W' \\ v_{-} = \mu_{-}E/p, \quad \alpha(E) = C_{1}p \exp[-D_{1}(p/E)^{\gamma_{2}}], & E/p < W \end{cases}$$
(2.9)

where $\mu_{-} = 3 \cdot 10^{5} \text{ cm}^{2} \cdot \text{mm Hg/V} \cdot \text{sec}$, $\mu_{+} = 10^{3} \text{ cm}^{2} \cdot \text{mm Hg/V} \cdot \text{sec}$, $k_{+} = 8.25 \cdot 10^{3} \text{ cm}^{2} \cdot \text{mm Hg/V} \cdot \text{sec}$, $W' = 60 \text{ V/cm} \cdot \text{mm Hg}$, $B_{1} = 2.2 \cdot 10^{-3} \text{ cm} \cdot \text{mm Hg/V}$, $B_{2} = 86.52 \text{ (mm Hg} \cdot \text{cm/V})^{-3/2}$, $C_{1} = 29.22 \text{ (cm} \cdot \text{mm Hg})^{-1}$, $D_{1} = 26.64 \text{ (V/cm} \cdot \text{mm Hg})^{1/2}$, and $W = 700 \text{ V/cm} \cdot \text{mm Hg}$.

The argon data was taken from [10]. In that paper, the quantity P_0 varied over the range $P_0 \sim 10^6 - 10^8$ R/sec. The system (2.1)-(2.5) in conjunction with Eqs. (2.8) or (2.9) completely defines the problem if the initial charge distribution and the value of the potential across the discharge gap are given. Since $v_+ \ll v_-$, electron inertia can be neglected by setting $\partial q_- / \partial t = 0$. Solving Eq. (2.1) by variation of the arbitrary constant, we obtain

$$i_{-}(x, t) = \exp\left\{\int_{0}^{t} \alpha(x', t) dx'\right\} \left[j_{-}\left(0, t\right) + Q\left(t\right)\int_{0}^{x} \exp\left\{-\int_{0}^{x} \alpha\left(x'', t\right) dx''\right\} dx'\right]$$
(2.10)

In order to determine the conditions for the applicability of the quasistationary equation (2.10), we compare it with the formal solution of Eq. (2.1) including electron inertia (under the condition $v_{-} = const$),

$$j_{-}(x, t) = \exp\left\{\int_{0}^{x} a\left\langle x', t - \frac{x - x'}{v_{-}}\right\rangle dx'\right\} \left[j_{-}\left(0, t - \frac{x}{v_{-}}\right) + \int_{0}^{x} Q\left\langle t - \frac{x - x'}{v_{-}}\right\rangle \exp\left\{-\int_{0}^{x'} a\left(x'', t - \frac{x - x''}{v_{-}}\right) dx''\right\} dx'\right]$$
(2.11)

It is then clear that Eq. (2.11) transforms into Eq. (2.10) if the functions change little during the time T_{-} .

3. We consider a finite-difference scheme for solution of the problem. We first consider the case of quasistationary electron equilibrium. We transform Eq. (2.10) into a difference equation by replacing integration with summation in accordance with the trapezoidal rule. Omitting intermediate transformations, we give an expression for Eq. (2.10):

$$j_{-}(m\Delta x, t) = j_{-}(0, t) Y (m\Delta x, t) + Q (t) Z (m\Delta x, t)$$

$$Y (m\Delta x, t) = \exp\left\{\sum_{k=0}^{m-1} \left[\alpha (k\Delta x, t) + \alpha ((k+1)\Delta x, t)\right]\right\}$$

$$Z (m\Delta x, t) = \frac{\Delta x}{2} \sum_{k=0}^{m-1} \exp\left\{\frac{\Delta x}{2} \sum_{i=1}^{m-1} \left[\alpha (i\Delta x, t) + \alpha ((i+1)\Delta x, t)\right]\right\} \left[1 + \exp\left\{-\frac{\Delta x}{2} \left[\alpha (k\Delta x, t) + \alpha ((k+1)\Delta x, t)\right]\right\}\right]$$

$$m = 1, 2, \dots, M; \quad M\Delta x = d$$
(3.1)

In finite differences, Eq. (2.2) is written in the form

$$\frac{q_{+}(x, t + \Delta t) - q_{+}(x, t)}{\Delta t} = \alpha j_{-}(x, t) + \frac{i_{+}(x + \Delta x, t) - j_{+}(x, t)}{\Delta x} + Q(t)$$

$$x = 0, \ \Delta x, \ 2\Delta x, \dots, (M-1)\Delta x; \quad t = 0, \ \Delta t, \ 2\Delta t$$
(3.2)

Similarly, by applying the trapezoidal rule to the boundary condition (2.4), we obtain

$$j_{+}(M\Delta x, t) = 0, \quad j_{-}(0, t) = \gamma_{i}j_{+}(0, t) + \gamma_{*}\mathfrak{s}(t)$$

$$\mathfrak{s}(t) = \frac{\Delta x}{2} \sum_{k=0}^{M-1} \left[\alpha(k\Delta x, t)j_{-}(k\Delta x, t) + \alpha((k+1)\Delta x, t)j_{-}((k+1)\Delta x, t) \right]$$
(3.3)

The representation of the Poisson equation and of the condition

$$\int E(x, t)dx = U$$

in finite differences is as described in [12]. We present the equation set. The field at the boundary x = 0 is given by

$$dE(0, t) = U - \frac{\Delta x}{2} \sum_{m=1}^{M} \left[E^*(m\Delta x, t) + E^*((m-1)\Delta x, t) \right] - \frac{\Delta x^2}{12} \left[q(M\Delta x, t) - q(0, t) \right]$$

$$E^*(m\Delta x, t) = 4\pi \sum_{k=1}^{m} \frac{\Delta x}{2} \left[q(k\Delta x, t) + q((k+1)\Delta x, t) \right], \ q \equiv q_- - q_+$$
(3.4)

From the known value E(0, t), the field within the gap is given as

$$E(m\Delta x, t) = E^*(m\Delta x, t) + E(0, t)$$
(3.5)

In order that the solution be stable when $\Delta x \rightarrow 0$, $\Delta t \rightarrow 0$, $\Delta t / \Delta x = \text{const}$, it is necessary to satisfy the inequality

$$\Delta t \leq \Delta x / v_{\star}$$

The system (3.1)-(3.5) describes a finite-difference scheme for quasistationary electron equilibrium. It is necessary to solve a system of $4(d/\Delta x + 1)$ equations in each time step in the case of an iterative solution. This can be simplified if two assumptions which introduce no noticeable error into the calculations [12] are made: use the quantity $q_{-}(x, t-\Delta t)$ on the right sides of Eqs. (3.4) in determining the field at the time t, and define the quantity $\sigma(t)$ in Eq. (3.3) as

$$\sigma(t) = \frac{\Delta x}{2} \sum_{k=0}^{M-1} \left[\alpha(k\Delta x, t) j_{-}(k\Delta x, t - \Delta t) + \alpha((k+1)\Delta x, t) \times j_{-}((k+1)\Delta x, t - \Delta t) \right]$$

If the field varies significantly during the time T_{-} , electron inertia should be included. It is necessary to consider in place of Eq. (2.10) a finite-difference representation of Eq. (2.1) having the form

$$q_{-}(k\Delta x, t + \Delta t) = \Delta t \left[\alpha \left(k\Delta x, t \right) j_{-}(k\Delta x, t) - \frac{j_{-}(k\Delta x, t) - j_{-}((k-1)\Delta x, t)}{\Delta x} + Q(t) \right] + q_{-}(k\Delta x, t) \quad k = 1, 2, \dots, M$$

The use of a program including electron inertia requires that the time step satisfy the condition $\Delta t \leq \Delta x/v_{-}$.

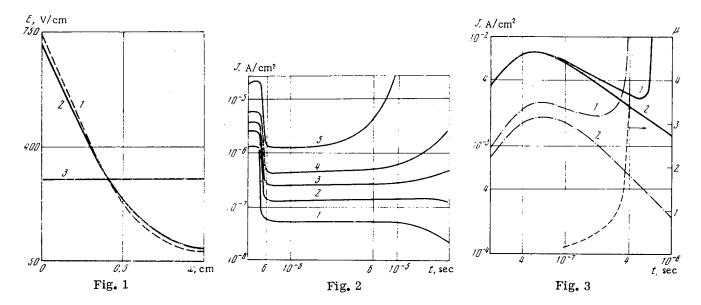
This leads to a considerable increase in the amount of machine time in comparison with the case of quasistationary electron equilibrium since $v_- \gg v_+$. An optimal arragement is the use of a program including electron inertia for times 0 < t < T, when the greatest rate of change of the field occurs, with a subsequent shift to a "quasistationary" program for t > T.

4. Such a program was used to calculate the transient current for various applied voltages U. If U is less than the breakdown voltage U_* , the current pulse J(t) created by an external effect dies out in time. When $U > U_*$, a sharp rise in current is evidenced in the transient current curve at a certain point in time (depending on the values of the coefficients α and γ and on the amplitude and duration of the ionizing pulse), which indicates the development in the system of an instability associated with the transition to a self-sustaining discharge. The limiting value of the parameter U which separates the rising and falling transient curves is equated to the dynamic breakdown voltage in this paper.

We consider the results of tests which were performed for the purpose of checking the correctness of the application of this algorithm to the calculation of the breakdown voltage.

The first set of calculations was related to the determination of the static volt-ampere characteristic of a gas in a stationary radiation field on the basis of a time-dependent system of equations. This problem can be solved if the ionizing pulse is given in the form of a rectangular step and the steady-state value of the current J corresponding to the initial parameter U is calculated.

This same problem can be solved using a time-independent system of equations. The resultant twocurrent boundary-value problem (of the two variable functions E(x) and $j_{-}(x)$, boundary conditions can be formulated only for the current $j_{-}(x)$ at the points x = 0 and x = d) is solved by the trial method using a



standard Runge-Kutta program in each breakdown version. The Runge-Kutta program provides considerably greater computational accuracy than that previously provided by the finite-difference scheme involving partial differential equations. It is convenient to take the total current J as a parameter because the equations are one-dimensional. The corresponding value of the potential is obtained by calculation of the field distribution and taking the integral

$$\int_{0}^{d} E(x) \, dx = U$$

We compare the field distribution curves obtained by the two methods. Figure 1 shows the field distribution at a point on the volt-ampere characteristic having the coordinates $J = 6 \cdot 10^{-6} \text{ A/cm}^2$, U = 300 Vin argon for p = 10 mm Hg, d = 1 cm, $\gamma_i = 0.02$, and $Q_0 = 10^{-6} \text{ C/cm}^3 \cdot \text{sec}$. The dashed curve 1 was obtained with the time-dependent algorithm and the solid curve 2 was obtained by application of the Runge-Kutta program to the stationary system; curve 3 gives the initial field distribution. The good agreement of the curves, which cover a nearly ninefold variation in the field, is evidence that the finite-difference representation of the Poisson equation and inclusion of the effect of field nonuniformity on avalanche development were done correctly in the time-dependent system.

The transient current was calculated for argon affected by a rectangular pulse of finite duration. The calculations were based on a quasistationary system with p = 10 mm Hg, d = 1 cm, $\gamma_i = 0.02$, $T = 0.5 \mu \text{sec}$, and $Q_0 = 10^{-6} \text{ C/cm}^3 \cdot \text{sec}$. Curves 1, 2, 3, 4, and 5 in Fig. 2 correspond to U = 300, 360, 390, 420, and 480 V. The sharp drop in current at $T = 0.5 \mu \text{sec}$ corresponds to the cutoff of the ionization pulse. Analysis of the curves for times greater than those shown in the figure indicate that a sharp rise in current is typical of curves 3, 4, and 5 while monotonic decay is typical of curves 1 and 2. The limiting value of U for which the nature of the time dependence of the current changes falls within the range 360 V < U < 390 V according to Fig. 2. If the breakdown voltage U_0 for this case is determined from the Townsend condition

$$\mu_0 = \gamma_i \left(\exp\left(\alpha_0 d\right) - 1 \right) = 1 \tag{4.1}$$

which transforms with the help of Eq. (2.9) to the form

$$U_0 = D_1^2 p d \ln^{-2} \{ C_1 p d / \ln \left(1 + 1 / \gamma_i \right) \}$$
(4.2)

it turns out to be 382 V. The calculated value agrees with the breakdown voltage obtained from the Townsend theory within the accuracy of the step $\Delta U = 30$ V by which the parameter U varies in the transition from one curve to the next. In the absence of space charge, the method assumed for the calculation of breakdown voltage gives a result which agrees with classical Townsend theory.

We present calculated results for the breakdown voltage of an air gap affect by a ionizing pulse of the form

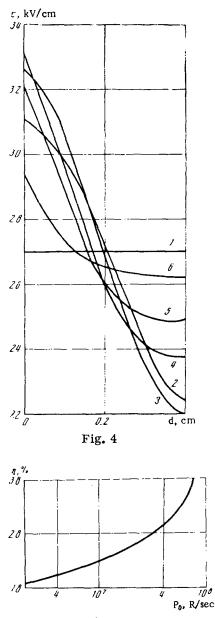


Fig. 5

$$P(t) = P_0 \left[\exp(-t / \tau_1) - \exp(-t / \tau_2) \right]$$
(4.3)

where $\tau_1 = 10^{-7}$ sec and $\tau_2 = 10^{-8}$ sec.

The pulse amplitude varied over the range $P_0 \sim 10^6 - 10^8$ R/sec. Two types of γ processes at the cathode were considered: emission of secondary electrons from the cathode because of ion impact, $\gamma = 0.02$; photoeffect at the cathode, $\gamma_* = 10^{-6}$. The calculation was made with a program which included electron inertia because the electric field varies greatly during the time T₋ in this case.

Figure 3 gives calculated curves for the transient current in air when p = 760 mm Hg, d = 0.4 cm, $\gamma_1 = 0.02$, $P_0 = 10^7 \text{ R/sec}$ (the dot-dashed curves 1 and 2 correspond to U = 11.1 and 10.8 kV), and $P_0 = 10^8 \text{ R/sec}$ (solid curves 1 and 2 correspond to U = 7.95 and 7.8 kV). It is clear that the breakdown voltage for $P_0 = 10^7 \text{ R/sec}$ is $U_* = 11.1$, and $U_* = 7.95$ for $P_0 = 10^8 \text{ R/sec}$ within the accuracy of the step $\Delta U = 0.3 \text{ kV}$. It is convenient to introduce the coefficient characterizing the relative reduction in breakdown voltage

$$\eta = (U_0 - U_*) / U_0$$

where U_0 is determined from the Townsend condition (4.1) with Eqs. (2.8) taken into account.

For the indicated values of the parameters p, d, and γ_i , we find $U_0 = 11.7$ keV and the coefficient $\eta = 32\%$ when $P_0 = 10^8$ R/sec. The significant reduction in breakdown voltage is explained by the strong nonuniformity of the field produced by the effect of the space charge. It is of interest to analyze the nature of the time variation of the field distribution. A set of curves characterizing the field distribution at different times is shown in Fig. 4. Curves 1, 2, 3, 4, 5, and 6 correspond to $t = 0, 0.335, 0.729, 2, 3, and 5 \mu sec$. The curves were obtained for $U < U_*$ (U = 10.8 kV, $P_0 = 10^7$ R/sec, dotdashed curve 2 in Fig. 3). Field nonuniformity increases, reaches a maximum, and then the field approaches the initial uniform distribution in proportion to the drop in current.

Analysis of the current curves for $U > U_*$ shows that the sharp rise in current is accompanied by a further increase in field nonuniformity. Similar transient current curves where photoeffect was present at the cathode with $w_{\rm c} = 10^{-6}$ (U₁ = 14.3 kV) were cal-

was present at the cathode with $\gamma_* = 10^{-6}$ (U₀ = 14.3 kV) were calculated for the following values of the amplitude of the ionizing pulse: P₀ = 10⁶, 10⁷, 5 \cdot 10⁷, and 8 \cdot 10⁷ R/sec. In accordance with the proposed technique, U_{*} was calculated for each value of P₀ and a curve of the reduction in breakdown voltage as a function of radiation dose rate was constructed (Fig. 5). A relative reduction in breakdown voltage amounting to 30-40% is observed for P₀ ~ 10⁷-10⁸ R/sec.

We consider whether or not the steep rise in the curves in Fig. 3 is a manifestation of the transition of the discharge into a self-sustaining mode. The variation of the coefficient $\mu(t)$ corresponding to the dot-dashed curve 1

$$\mu(t) = \gamma_i \left\{ \exp\left\{ \int_0^d \alpha \left[E(x, t) \right] dx \right\} - 1 \right\}$$

is shown in Fig. 3 (dashed curve). The coefficient $\mu(t)$ increases in proportion to the rise in current and reaches a value equal to one at the beginning of the sharp rise in current.

A second calculation of the transient current (corresponding to the dot-dashed curve 1 in Fig. 3) was made. The external ionizing pulse Q(t) was cut off at the time $t = 0.3 \mu sec$, which is before the sharp rise in the curve. Analysis of the results indicated the transient curve ended with a sharp rise in current in this

case. This is evidence that unstable growth of the current begins before the time when the transient curve undergoes a steep rise independently of the effect of external ionization.

In all the calculations, the step along the spatial coordinate was $\Delta x = 0.05d$; the time step was $\Delta t = 0.1\tau_2$ when working with the "inertial" program and was $\Delta t = 0.1T$ for a rectangular pulse or $\Delta t = 1 \mu \sec$ for rectangular step when working with the "quasistationary" program. To check that the results did not depend on the choice of Δx and Δt , trial calculations were made with smaller values of Δx and Δt .

Application of this algorithm to the calculation of the dynamic breakdown voltage in the case of an air gap showed that the breakdown voltage drops by nearly a factor of two for a sufficiently high radiation dose rate. The mechanism for the reduction in breakdown voltage advanced in this formulation of the problem is associated with the effect of the space charge produced by external ionization.

LITERATURE CITED

- 1. W. Rogowski and W. Fuchs, "Die Zundung einer bestrahlten Funkenstrecke," Arch. Elektrotekhnik, 29, No. 5 (1935).
- 2. W. Fuchs, "Gasentladung mit Fremdstrom aus der Kathode. Die Zundung im Raumladungsfeld," Z. Phys., 92, Nos. 7, 8 (1934).
- 3. W. Fuchs, "Zur Theorie der Zundspannungssenkung einer bestrahlten Funkenstrecke," Z. Phys., <u>98</u>, Nos. 11, 12 (1936).
- 4. E. M. Vol'f and V. S. Polikanov, "Theory of electrical conductivity of a gas ionized by an external source," Zh. Tekh. Fiz., <u>39</u>, No. 1, 152 (1969).
- 5. P. L. Auer, "Transient analysis of the Townsend discharge," Phys. Rev., 111, No. 3 (1958).
- 6. Y. Miyoshi, "Theoretical analysis of buildup of current in transient Townsend discharge," Phys. Rev., 103, No. 6 (1956).
- 7. G. Reter, Electron Avalanches and Gas Breakdown [Russian translation], Mir, Moscow (1968).
- 8. J. Pfaue, "Oszillographische Untersuchung der Stromanstiegs von Townsend-Entladungen im Raumladungsbereich," Z. Angew. Phys., <u>16</u>, No. 1 (1963).
- 9. W. Köhrmann, "Die zeitliche Entwicklung der Townsend-Entladung unter dem Einfluss der Raumladung," Z. Angew. Phys., 11, No. 11 (1959).
- 10. W. Borsch-Supan and H. Oser, "Numerical computation of the temporal development of current in a gas discharge tube," J. Res. Nat. Bur. Standards, Ser. B, 67, No. 1 (1963).
- 11. A. L. Ward, "Electronic computation of the temporal growth of current in a gas," Proc. Fifth Internat. Conf. on Ionization Phenomena in Gases, Munich, 1961, Vol. 2, North-Holland Publ. Co., Amsterdam (1972).
- 12. V. L. Granovskii, Electric Current in Gas [in Russian], Nauka, Moscow (1971).